REVISTA MEXICANA DE FÍSICA **44 SUPLEMENTO 1**, 93–95 SEPTIEMBRE 1998

Site order parameter p_g in $\pm J$ Ising square lattices

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Recibido el 12 de noviembre de 1997; aceptado el 14 de enero de 1998

Ising lattices with $\pm J$ exchange interactions present frustration that raises the energy of the ground level as well as its degeneracy. This leads to think that after going over the states in half the configuration space, the different ground states will end up alternating all spins yielding a null site order parameter. In the present work we give numerical results that indicate that there is always a large region in these lattices, which remains free of frustration in the ground state. So if the ergodic separation is done in such a way as lo leave this region with one possible spin orientation on one half the configuration space and the other spin orientation in the other half, it is possible to define a partial spin-glass phase with a positive site order parameter.

Keywords: Ising lattice; frustration; order parameter

Las redes de Ising con interacciones de intercambio $\pm J$ presentan frustración, lo que eleva la energía del fundamental aumentando grandemente su degeneración. Esto lleva a pensar que, al recorrer una mitad del espacio de configuración, los diferentes estados fundamentales terminarán por alternar todos los espines dando lugar a la existencia de un parámetro de orden de sitio nulo. En el presente trabajo entregamos resultados numéricos que indican que existe siempre una región grande en estas redes, la que permanece libre de frustración en el estado fundamental. De esta forma, si la separación ergódica se hace dejando una orientación de los espines en una mitad del espacio de configuración y la otra orientación en la otra mitad, es posible definir una fase parcial de vidrio de espín con un parámetro de orden de sitio positivo.

Descriptores: Red de Ising; frustración; parámetro de orden

PACS: 05.50.+q; 75.10.Hk; 75.10.Nr

Ising lattices with $\pm J$ exchange interactions cannot satisfy all local fields [1]. In each of the ground states a different combination of bonds are frustrated [2]. As we go over all the different ground states each site flips its spin in several occasions leading to a value zero for any site order parameter. However, if an ergodic separation is performed so only half the configuration space is retained, it is possible to think of a non zero order parameter.

In the present paper we want to investigate site order parameter p_g, which is defined as the fraction of sites that do not flip their spin when going over half of the ground states after we introduce an ergodic separation to be defined below. This site order parameter is more drastic than the site parameter q introduced by Edwards and Anderson, where flipping spins contribute to q with values between 0.0 and 1.0, going from a highly flipping spin to a frozen spin [3]. In our parameter p_g all such intermediate values are taken to be 0.0. Then, $q \leq p_g$.

In an early study we showed that when a random ergodic separation is performed $\langle p_g \rangle$ decreases very rapidly with the number of spins N or size of the system [4]. The average here is taken over a large number of randomly generated samples (500 in the present work). This means that in the thermodynamic limit p_g and q would be zero, so a spin-glass phase would not be possible for these lattices.

We would like to introduce here a different way of breaking ergodicity, which has more physical sense and leads to a non-vanishing order parameter p_g. To accomplish this let us first discuss the cause of the spin flips.

Due to the competing interactions in these $\pm J$ lattices not all the fields can be simultaneously satisfied. Namely, some of the bonds frustrate. If the ground state energy is preserved as a spin is flipped, frustration "jumps" to nearby bonds in what it is called itinerant frustration. [5]

When all bonds that frustrate at least once in the ground manifold are removed we obtain the *correlated diluted lattice* [6]. The ratio of the number of bonds in this diluted lattice (B_d) to the number of bonds in the original lattice (B), is defined as h_g [4, 7], the fractional content of bonds in the diluted lattice:

$$h_g = \frac{B_d}{B} = \frac{B_d}{2N}, \tag{1}$$

bearing in mind that $B = 2N$ for square lattices, which is the case we want to study here.

A careful scrutiny of the diluted lattice shows that the remaining bonds group themselves in regions or domains of connected bonds free of frustration [6]. A finer analysis shows that such regions prefer certain sizes (number of bonds in each region) and shapes [8]. However, the most striking feature is the existence of a dominant region, the largest one formed by B_L bonds. Lets us define the fractional bond con-

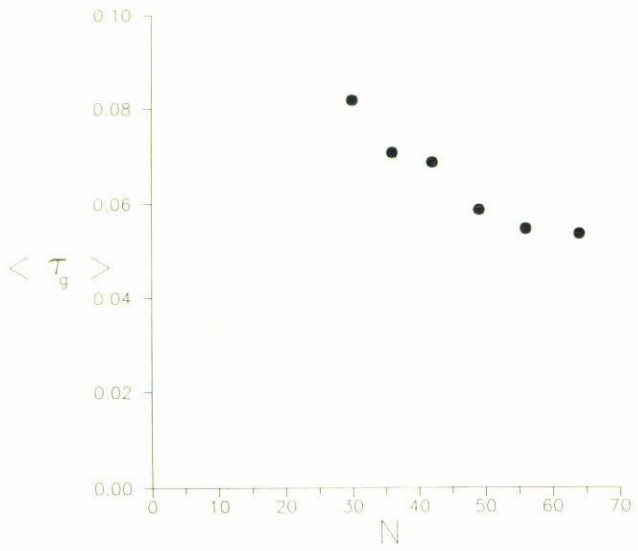

FIGURE 1. Fractional bond content of all but the largest region in the diluted lattice $\langle \tau_g \rangle$. Results correspond to average values over 500 samples for each different lattice size N reported.

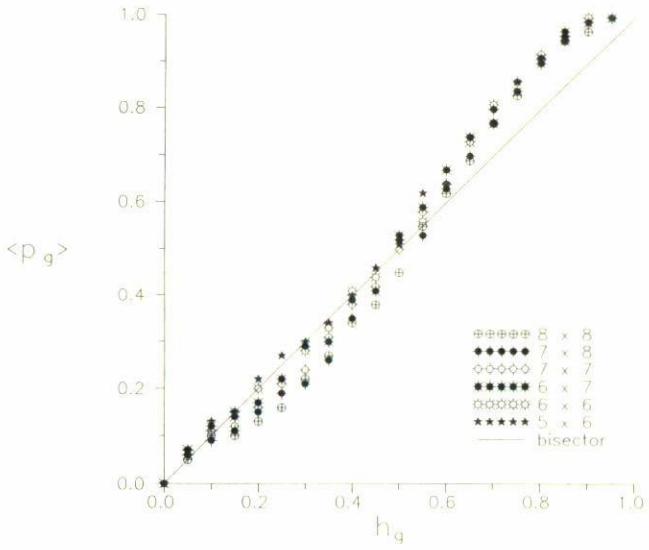

FIGURE 2. Site order parameter $\langle p_g \rangle$ as function of the fractional bond content of the diluted lattice h_g. The average values reported for each lattice size are taken within each channel in the h_g axis.

tent of the largest island t_g as,

$$t_g = \frac{B_L}{B}. \tag{2}$$

Numerical simulations show that $\langle t_g \rangle \approx 0.8 \langle h_g \rangle$, which means that most of the diluted lattices is given by the largest region [8]. The average here correspond to 500 samples for each of several lattice sizes. To investigate this dominance in a closer way, let us define the average size of the remaining regions, τ_g:

$$\tau_g = \frac{h_g - t_g}{\nu - 1}, \quad \nu > 1, \tag{3}$$

where ν is the number of regions in the diluted lattice. In the case $\nu = 1$, there is only one region so it does not make sense to define τ_g.

In Fig. 1 we present results for $\langle \tau_g \rangle$ for lattices of sizes $5 \times 6, 6 \times 6, 6 \times 7, 7 \times 7, 7 \times 8$, and 8×8. We report average values over 500 randomly generated samples for each size. Although $\langle \tau_g \rangle$ decreases rapidly with size for small values of N, this behavior stops very quickly with a clear indication for saturation at about $\langle \tau_g = 0.05 \rangle$ as N gets larger. It follows that the existence of one dominant region and many very small regions is a fact in these lattices.

Ergodic separation can be now defined in a unique way by anchoring on the dominant region. Since all its bonds are non frustrated, the connected spins remain always solidly linked to each other in the ground state. That is to say, this region need to be completely overturned in order to reach the mirror states in the configuration space, which is extremely costly in terms of energy. So we just pick any of the two spin orientations of the dominant region to break ergodicity. The immediate consequence of this is that the number of spin sites in

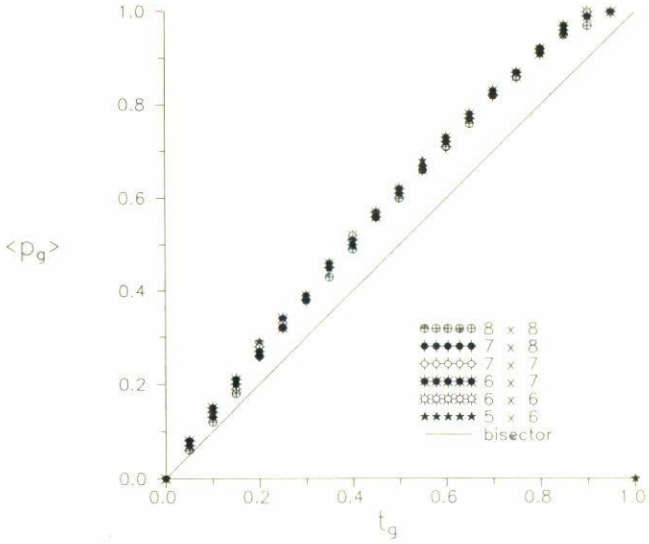

FIGURE 3. Site order parameter $\langle p_g \rangle$ as function of the fractional bond content of the largest region in the diluted lattice t_g. The average values reported for each lattice size are taken within each channel in the t_g axis.

the largest region n_L will contribute with a 1.0 to the order parameter p_g while all the remaining spins will not contribute since all the smallest regions will be overturned when scanning half of the configuration space. Namely,

$$p_g = \frac{n_L}{N}. \tag{4}$$

In Fig. 2 we report $\langle p_g \rangle$ for different channels within the natural distribution of h_g values. All the 500 samples generated for each lattice size are considered in this distribution. The main tendency is clear: $\langle p_g \rangle \approx h_g$. This means that the non–vanishing site order parameter is strongly coupled

to the unfrustrated regions in the diluted lattice. However, $\langle p_g \rangle \leq h_g$ for h_g under 0.45, for samples where there is no clear largest unfrustrated region. But for h_g over 0.45, it is evident that one large region takes over and $\langle p_g \rangle > h_g$.

Last comment is verified in Fig. 3, where we plot $\langle p_g \rangle$ *vs.* t_g. Again we define channel over the independent variables as done in Fig. 2. Clearly we find $\langle p_g \rangle > t_g$, except at the two extreme points. This result is consistent with the idea that the only contributions to p_g that are not canceled by the corresponding mirror states, are those coming from the largest region, the one that has been used to break ergodicity.

So as far as there exists a large region as $N \to \infty$, site order parameter $\langle p_g \rangle$ would not go to 0 if ergodicity is broken by considering one of the two orientations of such large region.

Acknowledgments

Authors acknowledge partial support from Dirección de Investigación de la Universidad de La Frontera under contract No. 96.29. Other author (EEV) acknowledges support from FONDECYT(Chile) under contract No. 1960972.

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